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On the volume and Gauss map image of spacelike submanifolds in de Sitter space form

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Abstract

In this paper we use Gauss map to study spacelike submanifolds in de Sitter space form. We prove that if there exist $\rho > 0$ and a fixed unit simple $(n+1)$-vector $a \in G^p_{n+1,p}$ such that the Gauss map g of an n-dimensional complete and connected spacelike submanifold M^n in S^{n+p}_p satisfies $\langle g, a\rangle \le \rho$, then M^n is diffeomorphic to S^n, and its volume satisfies $\mathrm{vol}(S^n)/\rho \le \mathrm{vol(M)} \le \rho^n \mathrm{vol}(S^n)$. We also characterize the case where these inequalities become equalities.

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1. The main result

Let R^{n+p+1}_p be the $(n+p+1)$-dimensional connected pseudo-Euclidean space with index p, that is, the real vector space R^{n+p+1} endowed with the pseudo-Euclidean metric

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tensor $\langle , \rangle$ given by

$$\langle v, w\rangle = \sum_{i=1}^{n+1} v_i w_i - \sum_{\alpha=n+2}^{n+p+1} v_\alpha w_\alpha,$$

and let S_p^{n+p} be the $(n+p)$-dimensional de Sitter space form with index p, that is,

$$S_p^{n+p} = \{x \in R_p^{n+p+1} : \langle x, x\rangle = 1\}.$$

A smooth immersion $\psi : M^n \to S_p^{n+p}$ of an n-dimensional connected manifold M^n is said to be a spacelike submanifold if the induced metric via ψ is a Riemannian metric on M^n. As is usual, the spacelike submanifold is said to be complete if the Riemannian induced metric is a complete metric on M^n.

Recently Aledo and Alías [1] used Gauss map image to study the topology and volume of complete spacelike hypersurfaces in de Sitter space and proved that a complete spacelike hypersurface M^n in de Sitter space whose image under the Gauss map is contained in a hyperbolic geodesic ball of radius ϱ is necessarily compact and its volume satisfies $\mathrm{vol}(S^n)/\cosh\varrho \leq \mathrm{vol}(M^n) \leq \mathrm{vol}(S^n)\cosh^n\varrho$. They also characterized the case where these inequalities become equalities. The main aim of this paper is to consider the similar problem in higher codimension. More precisely, we shall prove the following.

Main Theorem. *Let $\psi : M^n \to S_p^{n+p} \subset R_p^{n+p+1}$ be an n-dimensional complete spacelike submanifold in de Sitter space form S_p^{n+p}. If there exist $\rho > 0$ and a fixed unit simple $(n+1)$-vector $a \in G_{n+1,p}^p$ such that the Gauss map g of ψ satisfies $\langle g, a\rangle \leq \rho$, then M^n is diffeomorphic to S^n, and the volume of M^n satisfies*

$$\mathrm{vol}(S^n)/\rho \leq \mathrm{vol}(M^n) \leq \rho^n \mathrm{vol}(S^n). \tag{1}$$

Moreover, $\mathrm{vol}(M^n) = \rho^n\mathrm{vol}(S^n)$ *if and only if $\psi(M^n)$ is a totally umbilical n-sphere with radius ρ, while* $\mathrm{vol}(M^n) = \mathrm{vol}(S^n)/\rho$ *if and only if $\rho = 1$, and $\psi(M^n)$ is a totally geodesic n-sphere.*

We shall prove the Main Theorem in Section 3, here we give some simple applications as following.

Corollary 1. *The only complete spacelike surfaces* $(n = 2)$ *with parallel mean curvature in de Sitter space S_p^{2+p} whose Gauss map is bounded are the umbilical 2-spheres.*

Proof. From the Main Theorem we know that the surface is in fact compact. The result then follows from Theorem 5.3 in [2] or Corollary 9 in [3]. □

Corollary 2. *Let M^n be a complete spacelike n-submanifold with parallel mean curvature in de Sitter space S_p^{n+p} whose Gauss map is bounded. If the normal connection of M^n is flat, then M^n is a totally umbilical n-sphere.*

Proof. From the Main Theorem we know that the submanifold is in fact compact. The result then follows from Theorem 3 in [3]. □

2. The geometry of pseudo-Grassmannian

In this section we review some basic properties about the geometry of pseudo-Grassmannian. For details one is referred to see [5,6].

Let R_p^{n+p+1} be the $(n+p+1)$-dimensional pseudo-Euclidean space with index p, where, for simplicity, we assume that $n \geq p$. The case $n < p$ can be treated similarly. We choose a pseudo-Euclidean frame field $\{e_1, \dots, e_{n+p+1}\}$ such that the pseudo-Euclidean metric of R_p^{n+p+1} is given by $ds^2 = \sum_i (\omega_i)^2 - \sum_\alpha (\omega_\alpha)^2 = \sum_A \varepsilon_A (\omega_A)^2$, where $\{\omega_1, \dots, \omega_{n+p+1}\}$ is the dual frame field of $\{e_1, \dots, e_{n+p+1}\}$, $\varepsilon_i = 1$ and $\varepsilon_\alpha = -1$. Here and in the following we shall use the following convention on the ranges of indices:

$$1 \leq i, j, \dots \leq n+1; \qquad n+2 \leq \alpha, \beta, \dots \leq n+p+1;$$
$$1 \leq A, B, \dots \leq n+p+1.$$

The structure equations of R_p^{n+p+1} are given by

$$\mathrm{d}e_A = -\sum_B \varepsilon_A \omega_{AB} e_B,$$

$$\mathrm{d}\omega_A = -\sum_B \varepsilon_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0,$$

$$\mathrm{d}\omega_{AB} = -\sum_C \varepsilon_C \omega_{AC} \wedge \omega_{CB}.$$

Let $G_{n+1,p}^p$ be the pseudo-Grassmannian of all spacelike $(n+1)$-subspace in R_p^{n+p+1}, and $\tilde{G}_{n+1,p}^p$ the pseudo-Grassmannian of all timelike p-subspace in R_p^{n+p+1}. They are specific Cartan–Hadamard manifolds, and the canonical Riemannian metric on $G_{n+1,p}^p$ and $\tilde{G}_{n+1,p}^p$ is

$$ds_G^2 = ds_{\tilde{G}}^2 = \sum_{\alpha,i} (\omega_{\alpha i})^2.$$

Let 0 be the origin of R_p^{n+p+1}. Let $SO^0(n+p+1, p)$ denote the identity component of the Lorentzian group $O(n+p+1, p)$. $SO^0(n+p+1, p)$ can be viewed as the manifold consisting of all pseudo-Euclidean frames $(0; e_i, e_\alpha)$, and $SO^0(n+p+1, p)/SO(n+1) \times SO(p)$ can be viewed as $G_{n+1,p}^p$ or $\tilde{G}_{n+1,p}^p$. Any element in $G_{n+1,p}^p$ can be represented by a unit simple $(n+1)$-vector $e_1 \wedge \cdots \wedge e_{n+1}$, while any element in $\tilde{G}_{n+1,p}^p$ can be represented by a unit simple p-vector $e_{n+2} \wedge \cdots \wedge e_{n+p+1}$. They are unique up to an action of $SO(n+1) \times SO(p)$. The Hodge star operator "*" provides a one to one correspondence between $G_{n+1,p}^p$ and $\tilde{G}_{n+1,p}^p$. The product $\langle , \rangle$ on $G_{n+1,p}^p$ for $e_1 \wedge \cdots \wedge e_{n+1},\ v_1 \wedge \cdots \wedge v_{n+1} \in$

$G^p_{n+1,p}$ is defined by

$$\langle e_1 \wedge \cdots \wedge e_{n+1}, v_1 \wedge \cdots \wedge v_{n+1} \rangle = \det(\langle e_i, v_j \rangle).$$

The product on $\tilde{G}^p_{n+1,p}$ can be defined similarly.

Now we fix a standard pseudo-Euclidean frame $\{e_i, e_\alpha\}$ for R^{n+p+1}_p, and take $g_0 = e_1 \wedge \cdots \wedge e_{n+1} \in G^p_{n+1,p}$, $\tilde{g}_0 = *g_0 = e_{n+2} \wedge \cdots \wedge e_{n+p+1} \in \tilde{G}^p_{n+1,p}$. Then we can span the spacelike $(n+1)$-subspace g in a neighborhood of g_0 by $n+1$ spacelike vectors f_i:

$$f_i = e_i + \sum_\alpha z_{i\alpha} e_\alpha,$$

where $(z_{i\alpha})$ are the local coordinates of g. By an action of $SO(n+1) \times SO(p)$ we can assume that

$$(z_{i\alpha}) = \begin{pmatrix} \mu_1 & & \\ & \ddots & \\ & & \mu_p \\ & 0 & \end{pmatrix}.$$

From [6] we know that the normal geodesic $g(t)$ between g_0 and g has local coordinates

$$(z_{i\alpha}(t)) = \begin{pmatrix} \tanh(\lambda_1 t) & & \\ & \ddots & \\ & & \tanh(\lambda_p t) \\ & 0 & \end{pmatrix}$$

for real numbers $\lambda_1, \lambda_2, \ldots, \lambda_p$ such that $\sum_{i=1}^p \lambda_i^2 = 1$. This means that $g(t)$ is spanned by

$$f_1(t) = e_1 + \tanh(\lambda_1 t) e_{n+2}, \ldots, \; f_p(t) = e_p + \tanh(\lambda_p t) e_{n+p+1},$$
$$f_{p+1} = e_{p+1}, \ldots, \; f_{n+1} = e_{n+1}.$$

Consequently, $g(t)$ can also be represented by a unit simple $(n+1)$-vector as following:

$$g(t) = (\cosh(\lambda_1 t) e_1 + \sinh(\lambda_1 t) e_{n+2}) \wedge \cdots \wedge (\cosh(\lambda_p t) e_p + \sinh(\lambda_p t) e_{n+p+1}) \wedge e_{p+1} \wedge \cdots \wedge e_{n+1}.$$

Set $\lambda_\alpha = \lambda_{\alpha-n-1}$, then it is clear that $\cosh(\lambda_1 t) e_1 + \sinh(\lambda_1 t) e_{n+2}, \ldots, \cosh(\lambda_p t) e_p + \sinh(\lambda_p t) e_{n+p+1}, e_{p+1}, \ldots, e_{n+1}, \sinh(\lambda_{n+2} t) e_1 + \cosh(\lambda_{n+2} t) e_{n+2}, \ldots, \sinh(\lambda_{n+p+1} t) e_p + \cosh(\lambda_{n+p+1} t) e_{n+p+1}$ is again a pseudo-Euclidean frame for R^{n+p+1}_p, so we have

$$\tilde{g}(t) = *g(t) = (\sinh(\lambda_{n+2} t) e_1 + \cosh(\lambda_{n+2} t) e_{n+2})$$

$$\wedge \cdots \wedge (\sinh(\lambda_{n+p+1}t)e_p + \cosh(\lambda_{n+p+1}t)e_{n+p+1}) \in \tilde{G}^p_{n+1,p}.$$

Thus we have

$$\langle g_0, g(t)\rangle = (-1)^p\langle *g_0, *g(t)\rangle = (-1)^p\langle \tilde{g}_0, \tilde{g}(t)\rangle = \prod_\alpha \cosh(\lambda_\alpha t).$$

3. The proof of theorem

We shall complete the proof of theorem in this section. Let $\psi : M^n \to S^{n+p}_p \subset R^{n+p+1}_p$ be an n-dimensional complete spacelike submanifold in de Sitter space form S^{n+p}_p, and $e_1, \ldots, e_{n+p+1}$ the local pseudo-Euclidean frame field of R^{n+p+1}_p along ψ such that, when restricted on M^n, $e_1, \ldots, e_n$ are tangent to M^n and $e_{n+1} = \psi$ is the position vector of M^n. The Gauss map $g : M^n \to G^p_{n+1,p}$ is locally defined by $g = e_1 \wedge \cdots \wedge e_{n+1}$. In the following we shall also consider the map $\tilde{G} = *g = e_{n+2} \wedge \cdots \wedge e_{n+p+1} : M^n \to \tilde{G}^p_{n+1,p}$. Now we assume that there exist $\rho > 0$ and a unit simple $(n+1)$-vector $a = a_1 \wedge \cdots \wedge a_{n+1} \in G^p_{n+1,p}$ such that the Gauss map g satisfies $\langle g, a\rangle = \det(\langle e_i, a_j\rangle) \le \rho$. We extend $a_1, \ldots, a_{n+1}$ to a pseudo-Euclidean frame $a_1, \ldots, a_{n+p+1}$ of R^{n+p+1}_p, and define the projection $\Pi : M^n \to S^n_a$ by

$$\Pi(p) = \frac{1}{\sqrt{1 + \sum_\alpha \langle \psi(p), a_\alpha\rangle^2}}(\psi(p) + \sum_\alpha \langle \psi(p), a_\alpha\rangle a_\alpha), \forall p \in M^n, \tag{2}$$

where

$$S^n_a = \{x \in S^{n+p}_p : \langle x, a_\alpha\rangle = 0, n+2 \le \alpha \le n+p+1\}$$

is the totally geodesic n-sphere determined by a. A straightforward computation shows that

$$\begin{aligned}\mathrm{d}\Pi(X) = {} & \frac{1}{\sqrt{1 + \sum_\alpha \langle \psi, a_\alpha\rangle^2}}X + \frac{1}{(1 + \sum_\alpha \langle \psi, a_\alpha\rangle^2)^{3/2}} \\ & \times \left[\sum_\alpha (\langle X, a_\alpha\rangle + \sum_\beta \langle \psi, a_\beta\rangle^2 \langle X, a_\alpha\rangle - \sum_\beta \langle X, a_\beta\rangle\langle \psi, a_\beta\rangle\langle \psi, a_\alpha\rangle)a_\alpha \right. \\ & \left. - \sum_\alpha \langle \psi, a_\alpha\rangle\langle X, a_\alpha\rangle \psi \right] \end{aligned} \tag{3}$$

for every tangent field X on M^n, and consequently,

$$
\begin{aligned}
|\,\mathrm{d}\Pi(X)\,|^2 &= \frac{|\,X\,|^2}{1+\sum_\alpha \langle\psi, a_\alpha\rangle^2} + \frac{1}{(1+\sum_\alpha \langle\psi, a_\alpha\rangle^2)^2} \\
&\quad \times \left\{ \sum_\alpha \langle X, a_\alpha\rangle^2 + \sum_{\alpha,\beta} (\langle X, a_\alpha\rangle^2 \langle\psi, a_\beta\rangle^2 \right. \\
&\quad \left. - \langle X, a_\alpha\rangle\langle\psi, a_\alpha\rangle\langle X, a_\beta\rangle\langle\psi, a_\beta\rangle) \right\} \\
&= \frac{|\,X\,|^2}{1+\sum_\alpha \langle\psi, a_\alpha\rangle^2} + \frac{1}{(1+\sum_\alpha \langle\psi, a_\alpha\rangle^2)^2} \\
&\quad \times \left\{ \sum_\alpha \langle X, a_\alpha\rangle^2 + \sum_{\alpha<\beta} \langle X\wedge\psi, a_\alpha\wedge a_\beta\rangle^2 \right\}.
\end{aligned} \tag{4}
$$

From Section 2 we know that by an action of $SO(n+1)\times SO(p)$ we can assume that $e_\alpha = \cosh(\lambda_\alpha t)a_\alpha + \sinh(\lambda_\alpha t)a_{\alpha-n-1}$ so that

$$
\begin{aligned}
\langle g, a\rangle &= (-1)^p\langle\tilde{G}, \tilde{a}\rangle = (-1)^p\langle e_{n+2}\wedge\cdots\wedge e_{n+p+1}, a_{n+2}\wedge\cdots\wedge a_{n+p+1}\rangle \\
&= \prod_\alpha \cosh(\lambda_\alpha t),
\end{aligned} \tag{5}
$$

where $\sum_\alpha \lambda_\alpha^2 = 1$ and $t \in R$. Write

$$
a_\alpha = a_\alpha^T + \langle a_\alpha, \psi\rangle\psi - \sum_\beta \langle a_\alpha, e_\beta\rangle e_\beta,
$$

where a_α^T denotes the component of a_α which is tangent to M^n. Since a_α is a unit timelike vector for all α, we have

$$
-1 = |\,a_\alpha^T\,|^2 + \langle a_\alpha, \psi\rangle^2 - \sum_\beta \langle a_\alpha, e_\beta\rangle^2
$$

and so,

$$
1+\sum_\alpha \langle\psi, a_\alpha\rangle^2 = \sum_{\alpha,\beta}\langle a_\alpha, e_\beta\rangle^2 - \sum_\alpha |\,a_\alpha^T\,|^2 - p + 1 \le \sum_\alpha \cosh^2(\lambda_\alpha t) - p + 1. \tag{6}
$$

In order to estimate the quantity $\sum_\alpha \cosh^2(\lambda_\alpha t)$ we need the following.

Lemma. *Let $\mu_{n+2}\ge 1, \ldots, \mu_{n+p+1}\ge 1$ and $\prod_\alpha \mu_\alpha = C$. Then $\sum_\alpha \mu_\alpha^2 \le C^2 + p - 1$, and the equality holds if and only if $\mu_{\alpha_0} = C$ for some $n+2\le\alpha_0\le n+p+1$ and $\mu_\alpha = 1$ for any $\alpha\ne\alpha_0$.*

Proof. The lemma is trivial for $C = 1$. So we assume that $C > 1$. Without loss of generality, we can assume that $\mu_{n+2} \geq \cdots \geq \mu_{n+1+s} > 1$, $\mu_{n+2+s} = \cdots = \mu_{n+p+1} = 1$ and $\mu_{n+2} \cdots \mu_{n+1+s} = C$, here $1 \leq s \leq p$. Let us maximize

$$f(\mu_{n+2}, \ldots, \mu_{n+1+s}) = \sum_{\alpha=n+2}^{n+1+s} \mu_\alpha^2.$$

Let

$$F(\mu_{n+2}, \ldots, \mu_{n+1+s}, \mu) = \sum_{\alpha=n+2}^{n+1+s} \mu_\alpha^2 + \mu(C - \mu_{n+2} \cdots \mu_{n+1+s}),$$

where μ is a Langrange multiplier. Then at a critical point of f one has

$$0 = F_{\mu_\alpha} = 2\mu_\alpha - \mu\mu_{n+2} \cdots \mu_{\alpha-1}\mu_{\alpha+1} \cdots \mu_{n+1+s}, \quad n + 2 \leq \alpha \leq n + 1 + s$$

which implies that

$$2\mu_\alpha^2 = \mu\mu_{n+2} \cdots \mu_{n+1+s} = C\mu.$$

Thus at the critical point one has $\mu_{n+2} = \cdots = \mu_{n+1+s} = C^{1/s}$, and the corresponding critical value of f is $s \cdot C^{2/s}$. It is easy to see from mathematical analysis that $s \cdot C^{2/s} + p - s$ is monotone decreasing in s, so we have $\sum_\alpha \mu_\alpha^2 \leq C^2 + p - 1$, and the equality holds if and only if $\mu_{n+2} = C$, $\mu_{n+3} = \cdots = \mu_{n+p+1} = 1$. $\square$

Now from lemma, (4)–(6) and the assumption that $\langle g, a\rangle \leq \rho$, we conclude that

$$| \mathrm{d}\Pi(X) |^2 \geq \frac{| X |^2}{\langle g, a\rangle^2} \geq \frac{| X |^2}{\rho^2}. \tag{7}$$

It follows from (7) that Π is a local diffeomorphism. Since $\langle, \rangle$ is a complete Riemannian metric on M^n, the same holds for the homothetic metric $\widetilde{\langle, \rangle} = \langle, \rangle/\rho^2$. Then, (7) means that the map

$$\Pi : (M^n, \widetilde{\langle, \rangle}) \to (S_a^n, \langle, \rangle)$$

increases the distance. If a map, from a connected complete Riemannian manifold M_1 into another Riemannian manifold M_2 of the same dimension, increases the distance, then it is a covering map and M_2 is complete [4, VIII, Lemma 8.1]. Hence Π is a covering map, but S_a^n being simply connected this means that Π is in fact a global diffeomorphism between M^n and S_a^n. Hence, M^n is diffeomorphic to S^n.

Now we want to prove (1). Using the diffeomorphism $\Pi : M^n \to S_a^n$ we know that

$$\mathrm{vol}(S^n) = \int_{S_a^n} \mathrm{d}S = \int_{M^n} \Pi^*(\mathrm{d}S), \tag{8}$$

where $\mathrm{d}S$ stands for the volume element of S_a^n. From (2) and (3) it follows that

$$\begin{aligned}
&\Pi^*(\mathrm{d}S)(X_1, \ldots, X_n)\\
&\quad = \det(\mathrm{d}\Pi(X_1), \ldots, \mathrm{d}\Pi(X_n), a_{n+2}, \ldots, a_{n+p+1}, \Pi)\\
&\quad = \frac{1}{(1+\sum_\alpha \langle \psi, a_\alpha\rangle^2)^{(n+1)/2}} \det(X_1, \ldots, X_n, a_{n+2}, \ldots, a_{n+p+1}, \psi)\\
&\quad = \frac{(-1)^p \langle \tilde{g}, \tilde{a}\rangle}{(1+\sum_\alpha \langle \psi, a_\alpha\rangle^2)^{(n+1)/2}} \det(X_1, \ldots, X_n, e_{n+2}, \ldots, e_{n+p+1}, \psi)\\
&\quad = \frac{\langle g, a\rangle}{(1+\sum_\alpha \langle \psi, a_\alpha\rangle^2)^{(n+1)/2}} \mathrm{d}M(X_1, \ldots, X_n)
\end{aligned}$$

for tangent vector fields $X_1, \ldots, X_n$ of M^n, where $\mathrm{d}M$ is the volume element of M^n. In other words,

$$\Pi^*(\mathrm{d}S) = \frac{\langle g, a\rangle}{(1+\sum_\alpha \langle \psi, a_\alpha\rangle^2)^{(n+1)/2}} \mathrm{d}M. \tag{9}$$

From (5), (6), (8), and (9) and the lemma we see that

$$\begin{aligned}
\mathrm{vol}(S^n) &= \int_{M^n} \frac{\langle g, a\rangle}{(1+\sum_\alpha \langle \psi, a_\alpha\rangle^2)^{(n+1)/2}} \mathrm{d}M \geq \int_{M^n} \frac{1}{\langle g, a\rangle^n} \mathrm{d}M \geq \int_{M^n} \frac{1}{\rho^n} \mathrm{d}M\\
&= \frac{1}{\rho^n} \mathrm{vol}(M^n),
\end{aligned}$$

and so $\mathrm{vol}(M^n) \leq \rho^n \mathrm{vol}(S^n)$, and if the equality holds, then $a_\alpha^T = 0$ for all α, thus $\langle \psi, a_\alpha\rangle =$ constant for all α. It is easy to see that M^n is a totally umbilical n-sphere with radius ρ. Conversely, if M^n is a totally umbilical n-sphere with radius ρ, we certainly have $\mathrm{vol}(M^n) = \rho^n \mathrm{vol}(S^n)$. Similarly,

$$\mathrm{vol}(S^n) = \int_{M^n} \frac{\langle g, a\rangle}{(1+\sum_\alpha \langle \psi, a_\alpha\rangle^2)^{(n+1)/2}} \mathrm{d}M \leq \int_{M^n} \langle g, a\rangle \mathrm{d}M \leq \rho \mathrm{vol}(M^n),$$

so $\mathrm{vol}(M^n) \geq \mathrm{vol}(S^n)/\rho$, and the equality holds if and only if $\langle \psi, a_\alpha\rangle = 0$ for all α and $\langle g, a\rangle = \rho$. Thus M^n is a totally geodesic n-sphere and $\rho = 1$, and the theorem is proved.

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